Year 13 Mathematics IAS 3.4

Critical Path Analysis

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Contents

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NCEA 3 Internal Achievement Standard 3.4 – Critical Path Anaysis

This achievement standard involves using critical path analysis in solving problems.

Achievement	Achievement with Merit	Achievement with Excellence
• Use critical path analysis in solving problems.	• Use critical path analysis with relational thinking, in solving problems.	• Use critical path analysis with extended abstract thinking, in solving problems.

- This achievement standard is derived from Level 8 of The New Zealand Curriculum and is related to the achievement objectives
  - develop network diagrams to find optimal solutions, including critical paths in the Mathematics strand of the Mathematics and Statistics Learning Area.

- Use critical path analysis in solving problems involves:
  - selecting and using methods
  - demonstrating knowledge of concepts and terms
  - communicating using appropriate representations.

- Relational thinking involves one or more of:
  - carrying out a logical sequence of steps
  - connecting different concepts or representations
  - demonstrating understanding of concepts
  - forming and using a model;

  and relating findings to a context, or communicating thinking using appropriate mathematical statements.

- Extended abstract thinking involves one or more of:
  - devising a strategy to investigate or solve a problem
  - identifying relevant concepts in context
  - developing a chain of logical reasoning, or proof
  - forming a generalisation;

  and using correct mathematical statements, or communicating mathematical insight.

- Problems are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. Situations will be set in real-life or mathematical contexts.

- Methods include a selection from those related to:
  - precedence tables
  - network diagrams
  - critical events
  - scheduling
  - float times.

Critical Path Analysis

Critical Path Analysis

In this Achievement Standard we focus on the modelling of procedures or projects.

This process of modelling is often called Network Analysis or Critical Path Analysis and involves the use of specific techniques for planning, scheduling, management and control of a project.

Critical Path Analysis enables us to break the project down into individual components or activities and then record the results in an appropriate network diagram.

Critical Path Analysis is a valuable management tool because it results in

- a clear identification of all major components or activities in a project.
- a logical sequencing of all components or activities.
- an allocation of time required for each component or activity.
- a scheduling of the activities or components in the most efficient manner.
- a visual picture in the way of a network diagram of the project
- the ability to evaluate the network in an ongoing manner.

Precedence Tables

One of the first steps in scheduling a project is to break it down into a series of components or activities and represent these in table form.

Such a table is called a Precedence table (or Dependence table) and it lists all activities required to complete a project with an applicable time (duration) alongside each activity. Included as well are any dependencies, i.e. when an activity depends on another being completed first.

Study the Precedence table drawn on the right for the 'Construction of a Garden Shed'.

The column headed Precedence ('Depends on') shows those activities that have to be completed before a particular one can be started. For example, the walls of the garden shed cannot be erected before the site is cleared, the foundations have been laid and the timber purchased, hence activity D has B and C in the precedence column alongside it.

The precedence column only shows the activities immediately preceding each entry in the activity column, but by implication these can include other activities.

This activity can be done without having to do any other activity before it.

This is the time (duration) the activity takes.

Activity and Description (Garden shed construction)		Precedence ('Depends on')	Duration (days)
A	Clear building site	–	1
B	Lay foundations	A	4
C	Purchase timber	–	1
D	Erect walls	B, C	3
E	Construct roof	D	2
F	Clad Walls	E	3
G	Install door and windows	F	1
H	Fit spouting	G	1
I	Paint	H	2

B, C in the Precedence column means that before the walls can be erected the foundations have to be laid and the timber purchased.

For example D depends on B and C, and in turn B depends on A.

So in order to erect the walls of the garden shed the site has to be cleared, the foundations laid and the timber purchased.

Consider cladding the walls of the garden shed which is F.

F depends on E, but since E depends on D it is implied that F depends on D. We already know that D depends on B and C and in turn B depends on A.

Achievement – Answer the following.

1. The project 'Bake a cake' can be broken into the following activities. The time in minutes to complete each activity is included in brackets.

   A Select required recipe (10).

   B Preheat the oven (15).

   C Find and grease required baking tin (2).

   D Get the ingredients (5).

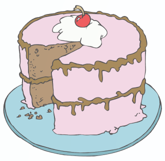
   E Measure and mix the ingredients (10).

   F Pour mixture into baking tin (1).

   G Bake cake in oven (25).

   H Place cake on tray to cool (45).

   Complete the precedence table below for baking a cake.



Activity and Description (Bake a cake)	Precedence ('Depends on')	Duration (mins.)
A Select recipe		10
B Preheat the oven		15
C Find and grease baking...		2
D Get ingredients		5
E Measure and mix ingre...		10
F Pour mixture into tin		1
G Bake cake in oven		25
H Place cake on tray to cool		45

2. A project has been split into six activities labelled A, B, C, D, E and F. The following information relates to the project.

   - Activity A, B and E are independent of the others.
   - Only when activity A is completed can C be started.
   - Activity A, B and C must be completed before activity D can be started.
   - Activity F is the final activity and cannot be started until all other activities are finished.

   Complete the precedence table top right for this project. Note, there is no duration column.

Activity	Precedence ('Depends on')
A	
B	
C	
D	
E	
F	

3. The project 'Make breakfast' which involves making a cup of tea and some toast can be broken into the following activities. The time in minutes to complete each activity is included in brackets.

   Note, the activities are in a random order.

   A Get up and dressed (15)

   B Eat toast and drink tea (20).

   C Get bread (1).

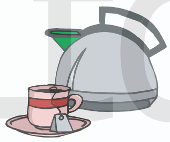
   D Fill kettle with water (1).

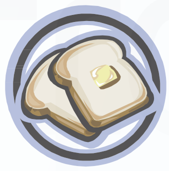
   E Make the tea (2).

   F Boil the kettle (3).

   G Put spread on toast (1).

   H Toast bread (3).



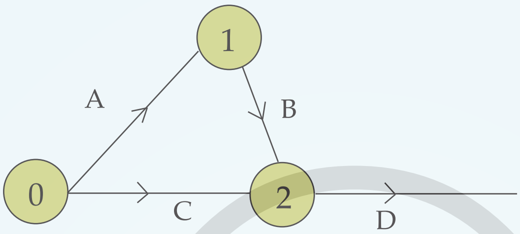


   Complete the precedence table below for making breakfast.

Activity and Description (Make breakfast)	Precedence ('Depends on')	Duration (mins.)
A Get up and dressed		
D Fill kettle with water		
F Boil the kettle		
E Make the tea		
C Get bread		
H Toast bread		
G Put spread on toast		
B Eat toast and drink tea		

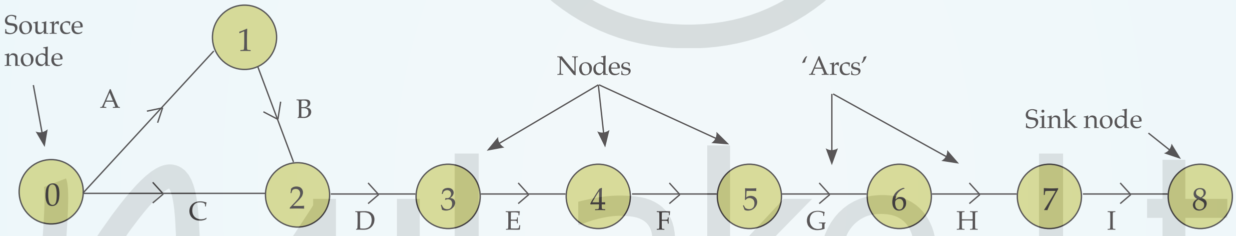
Network Diagrams cont...

- Next activities B and C must be completed before activity D, 'Erect walls' can be started. Therefore we insert node 2, which indicates the completion of activities B and C and add activity D.



Sometimes your first attempt at drawing a network diagram may not be perfect so it may be necessary to adjust it as you go. You can see above that we have had to adjust the shape of our network diagram in order to add activity D into the network diagram.

- The rest of the network diagram for the 'Construction of a shed' is straightforward as each remaining activity only depends on its predecessor, i.e. activity E depends on activity D, activity F depends on activity E, activity G depends on activity F, activity H depends on activity G and finally activity I depends on activity H. We can represent this by adding each remaining activity to the existing diagram. Our final network diagram can be seen below.

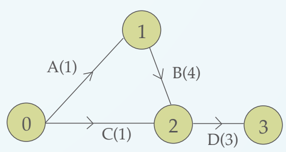


Once all the activities have been undertaken the project is complete. We add a finishing node (called a sink node), node 8 to indicate this.

Activity and Description (Garden shed construction)		Precedence ('Depends on')	Duration (days)
A	Clear building site	–	1
B	Lay foundations	A	4
C	Purchase timber	–	1
D	Erect walls	B, C	3
E	Construct roof	D	2
F	Clad Walls	E	3
G	Install door and windows	F	1
H	Fit spouting	G	1
I	Paint	H	2

On an activity network the activities are represented by lines ('arcs') and the weight on each line is the duration of the activity.

On our activity network above we have not included the duration, but will do so in the next section. When we do this we will include the duration in brackets e.g. A(1), B(4), C(1), D(3) etc.



The nodes (or vertices) on an activity network represent events, where each event represents the completion of one or more activities.

When we construct an activity network, convention is that we construct it from left to right.

The source node represents the start of a project and the sink node the end of the project.

An activity network consists of nodes and arcs. The nodes are represented by circles and the 'arcs' by lines between the nodes. A node indicates the completion of one or more activities.

Example

A precedence table with duration times (hours) for the activities required to complete a project is given below. Draw up an activity network to model this project and then undertake a forward and backward pass of the activity network.

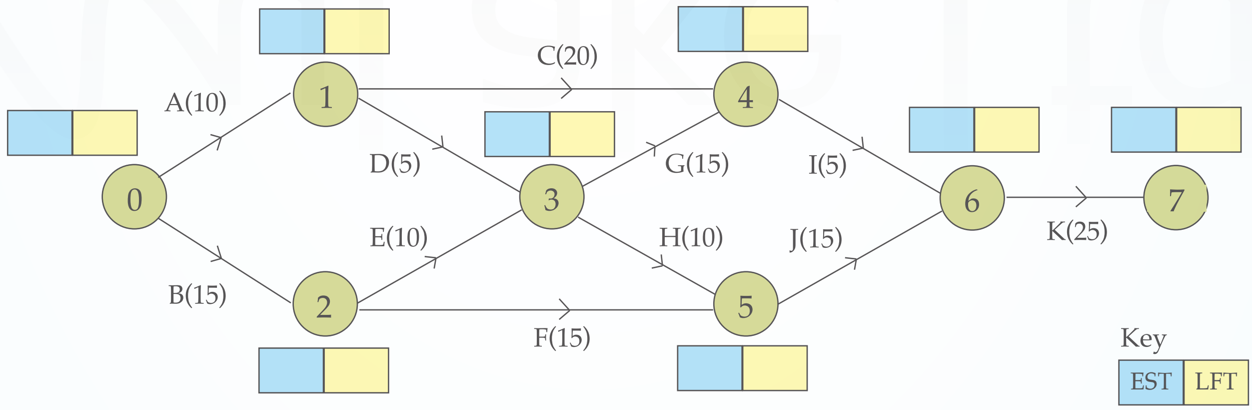
Activity	Precedence ('Depends on')	Duration (hours)
A	–	10
B	–	15
C	A	20
D	A	5
E	B	10
F	B	15
G	D, E	15
H	D, E	10
I	C, G	5
J	F, H	15
K	I, J	25

Extra Examples

When undertaking a forward pass remember you want the latest start time (LST).

Put your finger on a node and ask yourself what is the LST from the nodes that point to the node with your finger on.

We begin by drawing up the activity network for this precedence table. Note, we have included the duration times in brackets after each activity and have included a box alongside each node so we can enter in the earliest and latest event times when we undertake our forward and backward pass.



We now begin our **forward pass** (blue) of the activity network.

At node 0 we put the duration time of 0 in the left hand rectangle (blue).

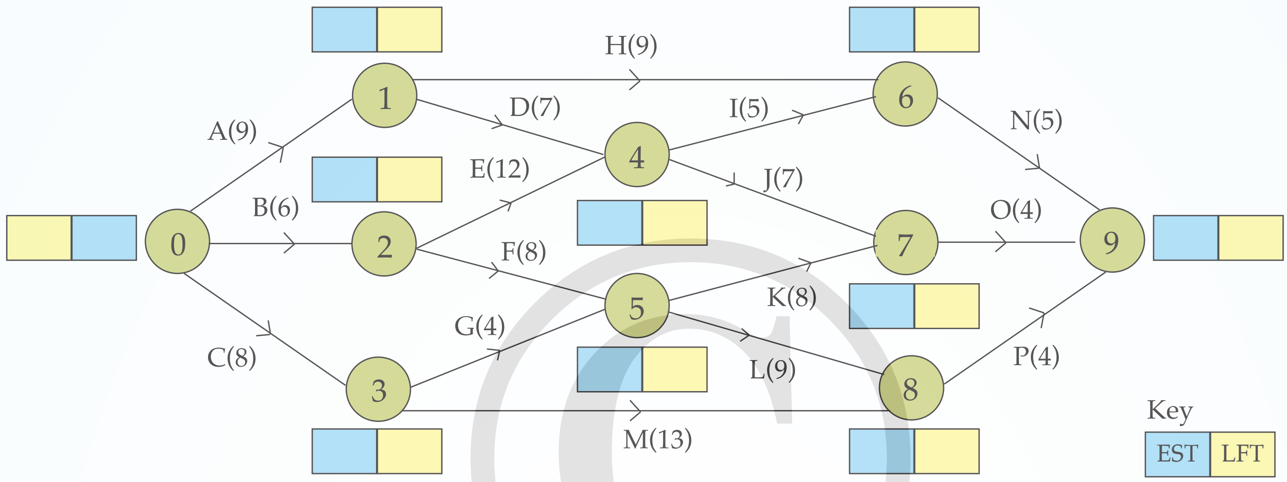
At node 1 the duration time must be 10 as it takes 10 hours to complete this activity.

At node 2 the duration time must be 15 as it takes 15 hours to complete this activity.

At node 3 we have two possibilities which we must calculate. The path 0 – 1 – 3 which gives a total of 15 hours and the path 0 – 2 – 3 which gives a total of 25 hours. The maximum of these two values is 25 which is the value we enter for node 3.

Achievement – For each of the activity networks below undertake a forward and backward pass to calculate the early and late times for each event. Determine the critical events and the critical path of the network.

22.



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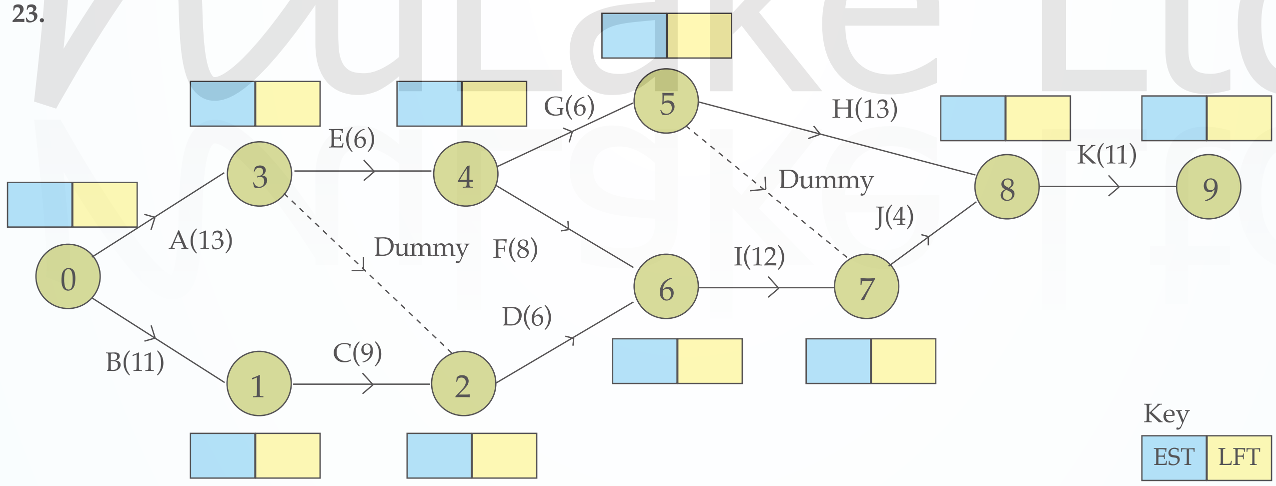


The network above includes two dummy activities. Remember that dummy activities have a duration of zero.

Scheduling cont...

The two key times associated with activity C are:
The earliest start time (EST = 10) and the latest finish time (LFT = 45).
We now calculate the earliest finish time and the latest start time

The earliest finish time for C EFT = EST + D
= 10 + 20
= 30.

The latest start time for C LST = LFT – D
= 45 – 20
= 25.

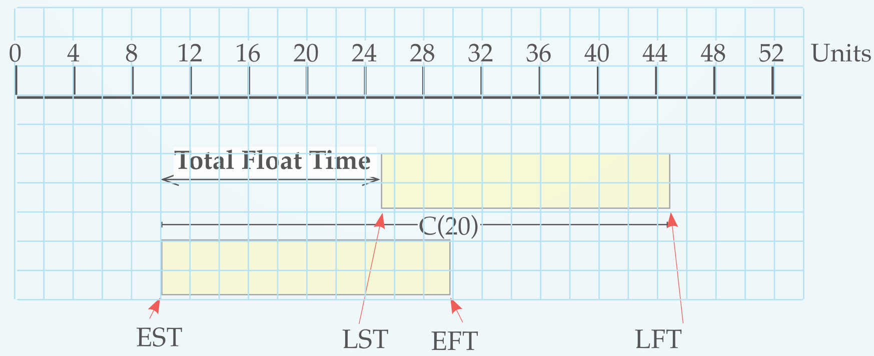
We now summarise these four times for each activity from our network diagram in a table.

Activity	Duration (D)	EST	EFT (EFT = EST + D)	LST (LST = LFT – D)	LFT	Total float time LFT – EST – D
A	10	0	10	10	20	10
B	15	0	15	0	15	0
C	20	10	30	25	45	15
D	5	10	15	20	25	10
E	10	15	25	15	25	0
F	15	15	30	20	35	5
G	15	25	40	30	45	5
H	10	25	35	25	35	0
I	5	40	45	45	50	5
J	15	35	50	35	50	0
K	25	50	75	50	75	0

Remember that the total float time is the latest finish time (LFT) – earliest start time (EST) – duration of the activity (D).

TFT = LFT – EST – D

Remember the total float time is the amount of time we can delay an activity by and not effect the latest finish time and hence the whole project's completion time.



We could also find the total float time by calculating TFT = LFT – EFT or TFT = LST – EST but as EFT (or LST) have to be calculated first it is usually easier to stick to our first formula.

TFT = LFT – EST – D

cont...

b) We now draw up a time schedule for the project given that each activity requires one worker and when we assign a worker, we do so to the activity that is the most critical, i.e. whose latest start time (LST) is the smallest. Also a worker must not remain inactive if there is an activity that can be started.

We begin by assigning the activities along the critical path (B, F, H and I) to worker 1 (see diagram below). A has a LST of 0 so we start there. There should be no gaps along the critical path as each activity has EST equal to the LFT of the previous activity.

The next activity with the smallest LST is activity A, with a LST of 8. We assign A to worker 2 and start them as soon as possible, i.e. 0 (this may change later).

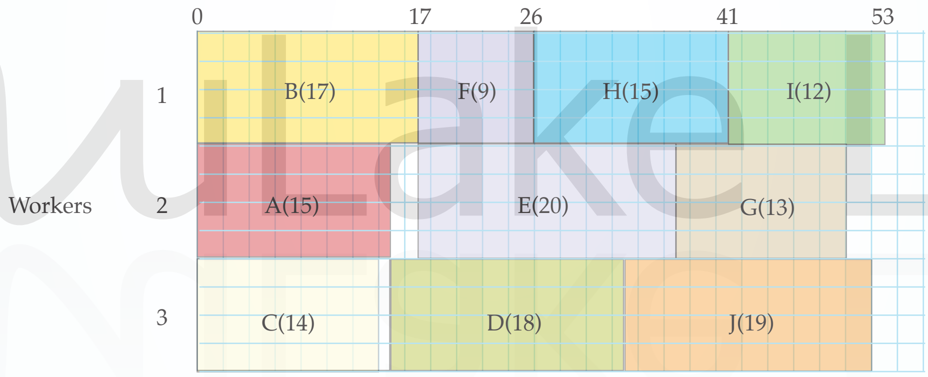
The next activity with the smallest LST is E with a LST of 21 and an EST of 17. We also assign this activity to worker 2 starting at 17, its EST.

The next activity with the smallest LST is D with a LST of 23 and an EST of 15. Since activity E (with worker 2) has an EFT of 37 we must assign activity D to worker 3 starting at 15, its EST.

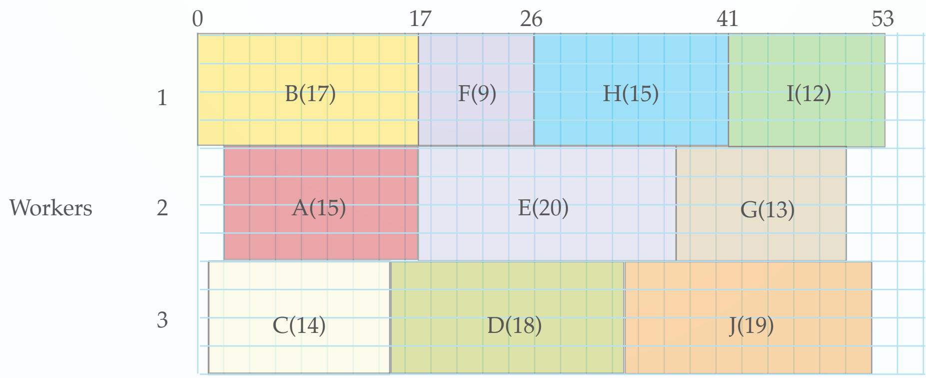
The next activity with the smallest LST is C with a LST of 26 and an EST of 0. Since activity C has an EFT of 14 and a LFT of 40 we cannot assign it to worker 2 so we assign it to worker 3 starting at 0, its EST (this may change later).

The next activity with the smallest LST is J with a LST of 34 and an EST of 26. Since activity E has an EFT of 37 and activity D has an EFT of 33 we assign activity G to worker 3 starting at 33 and finishing at 52.

The next activity with the smallest LST is G with a LST of 40 and an EST of 14. Since activity E has an EFT of 37 and activity J has a finishing time of 52 we assign activity G to worker 2 starting at 37.



Looking at the tiles in our diagram we can see that we can push A in 2 days to remove the gap between A and E. This is possible as A can start as late as 8 days. Similarly we can push C (LST = 26 days) in to remove the gap between C and D. C now starts at 1 day and finishes at 15 days.



c) Looking at our schedule (not unique) we note that the entire project will require a minimum of 3 workers in order to complete it in the 53 days project time. Worker 1 will work all 53 days, worker 2 will work 50 – 2 = 48 days and worker 3 will work 52 – 1 = 51 days.

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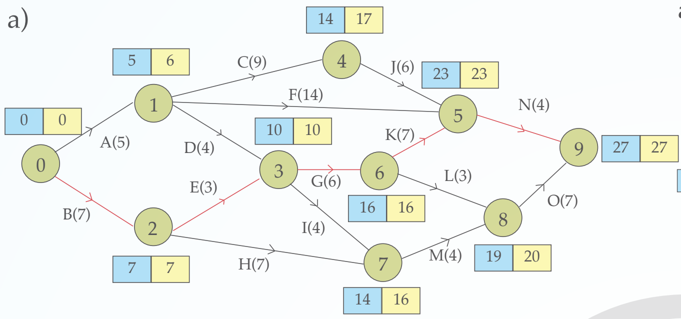
a)



Critical events: B, E, G, K, N.
Critical path: 0 – 2 – 3 – 6 – 5 – 9.

b)

Non-critical activities	Free float time	Total float time
A	0	1
C	0	3
D	1	1
F	4	4
H	0	2
I	0	2
J	3	3
L	0	1
M	1	2
O	1	1

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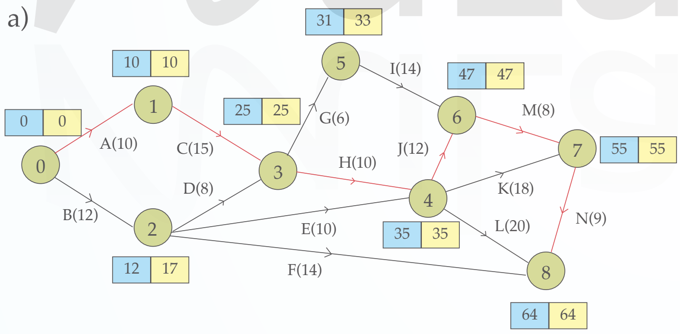
28.

a)



Critical events: A, C, H, J, M, N.
Critical path: 0 – 1 – 3 – 4 – 6 – 7 – 8.

b)

Non-critical activities	Free float time	Total float time
B	0	5
D	5	5
E	13	13
F	38	38
G	0	2
I	2	2
K	2	2
L	9	9

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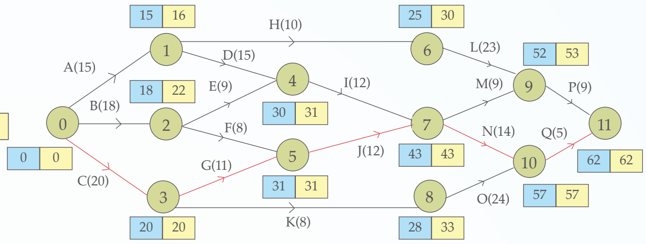
29.

a)



Critical events: C, G, J, N, Q.
Critical path: 0 – 3 – 5 – 7 – 10 – 11.

b)

Non-critical activities	Free float time	Total float time
A	0	1
B	0	4
D	0	1
E	3	4
F	5	5
H	0	5
I	1	1
K	0	5
L	4	5
M	0	1
O	5	5
P	1	1

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c) Critical events: B, F, J, N, Q.
Critical path: 0 – 2 – 5 – 7 – 10 – 11.
Extends the project time by 1 unit to 63 units.

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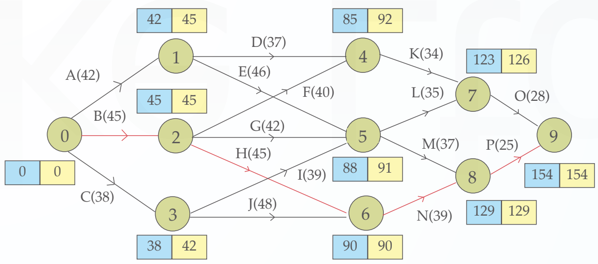
30.

a)



Critical events: B, H, N, P.
Critical path: 0 – 2 – 6 – 8 – 9.

b)

Non-critical activities	Free float time	Total float time
A	0	3
C	0	4
D	6	13
E	0	3
F	0	7
G	1	4
I	11	14
J	4	4
K	4	7
L	0	3
M	4	4
O	3	3